Sonoluminescence as quantum vacuum radiation

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Sonoluminescence is explained in terms of quantum radiation by moving interfaces between media of different polarizability. It can be considered as a dynamic Casimir effect, in the sense that it is a consequence of the imbalance of the zero-point fluctuations of the electromagnetic field during the non-inertial motion of a boundary. The transition amplitude from the vacuum into a two-photon state is calculated in a Hamiltonian formalism and turns out to be governed by the transition matrix-element of the radiation pressure. Expressions for the spectral density and the total radiated energy are given.

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Sonoluminescence is a phenomenon that has so far resisted all attempts of explanation. A short and intense flash of light is observed when ultrasound-driven air or other gas bubbles in water collapse. This process has been known for more than 60 years to occur randomly when degassed water is irradiated with ultrasound [1]. Recently interest has been revived by the contriving of stable sonoluminescence [2, 3] where a bubble is trapped at the pressure anti-node of a standing sound-wave in a spherical or cylindrical container and collapses and re-expands with the periodicity of the sound. With a clock-like precision a light pulse is emitted during every cycle of the sound-wave; the jitter in the sequence of pulses is almost unmeasurably small. Shining laser light upon the bubble and analyzing the scattered light on the basis of the Mie theory of scattering from spherical obstacles one has been able to record the time-dependence of the bubble radius [4]; these experiments showed that the flash of light is emitted shortly after the bubble has collapsed, i.e. shortly after it has reached its minimum radius. This and the fact that the spectrum of the emitted light resembles radiation from a black body at several tens of thousands degree Kelvin have led to the conjecture that the light could be thermal radiation from the highly compressed and heated gas contents of the bubble after the collapse [5]. It has also been argued that the experimentally observed spectrum would equally well be compatible with the idea of a plasma forming at the bubble centre after the collapse and radiating by means of bremsstrahlung [6]. An alternative suggestion has tried to explain the sonoluminescence spectrum as pressure-broadened vibration-rotation lines [7], but although this theory has been very successful in the case of randomly excited (multi-bubble) sonoluminescence seen in silicone oil it has been inefficacious for sonoluminescence in water.

All of the above theories have serious flaws. Both black-body radiation and bremsstrahlung would make a substantial part of the radiated energy appear below 200nm where the surrounding water would absorb it. If one estimates the total amount of energy to be absorbed corresponding to the observed number of photons above the absorption edge, one quickly convinces oneself that this would be far too much to leave no macroscopically discernible traces in the water, as for instance dissociation [8]; however, nothing the like is observed. Another very strong argument against all three of the above theories is that the processes involved in each of them are far too slow to yield pulse lengths of 10ps or less but which are observed. Moreover, if a plasma were formed in the bubble, one should see at least remnants of slow recombination radiation from the plasma when the bubble re-expands. As to the theory involving vibration-rotation excitations, the line-broadening required to model the observed spectrum seems rather unrealistic.

In its concept the theory to be presented here has been loosely inspired by Schwinger's idea [9] that sonoluminescence might be akin to the Casimir effect in the sense that the zero-point fluctuations of the electromagnetic field might lie at the origin of the observed radiation. More closely related to this is the Unruh effect well-known in field theory [10]; its original statement is that a uniformly accelerated mirror in vacuum emits photons with the spectral distribution of black-body radiation. However, the phenomenon is far more general than that and in particular not restricted to perfect mirrors. This kind of quantum vacuum radiation has been shown to be generated also by moving dielectrics [11]. Whenever an interface between two dielectrics or a dielectric and the vacuum moves non-inertially photons are created. In practice this effect is very feeble, so that it has up to now been far beyond any experimental verification. Sonoluminescence might be the first identifiable manifestation of quantum vacuum radiation.

The mechanism by which radiation from moving dielectrics and mirrors in vacuum is created is understood most easily by picturing the medium as an assembly of dipoles. The zero-point fluctuations of the electromagnetic field induce these dipoles and orient and excite them.

However, as long as the dielectric stays stationary or uniformly moving such excitations remain virtual; real photons are created only when the dielectric or mirror moves non-uniformly, because then the fluctuations get out of balance and no longer average to zero. Mechanical energy of the motion of the dielectric is dissipated into the field and a corresponding frictional force is felt by the dielectric. The fluctuation-dissipation theorem predicts this frictional force [12] in terms of the force fluctuations on the stationary dielectric or mirror [13]; it holds, however, no information on the state of the photon field, i.e. the radiated spectrum cannot be evaluated from the fluctuation-dissipation theorem.

The surface of an air bubble in water is such an interface between two dielectric media. When the bubble collapses, the motion of the interface is highly non-linear; the acceleration and higher derivatives of the velocity attain values that are high enough to make quantum vacuum radiation a non-negligible process.

The present model describes the bubble as a spherical cavity in a uniform dielectric medium. The refractive index of water is roughly 1.3 in the visible spectrum, and the air inside the bubble has a refractive index of practically 1 even if strongly compressed. The assumption of the uniformity of the water is of course unrealistic, but the variation of the refractive index in the vicinity of the bubble surface is of secondary importance for the problem of vacuum radiation. For the present purposes the bubble can to a very good approximation be described by a step in the dielectric function

$$\varepsilon(r; R) = 1 + (n^2 - 1)\,\theta(r - R)\;. \tag{1}$$

Here n is the refractive index of water which is for simplicity assumed to be constant and non-dispersive, and the refractive index of the bubble contents has been set to 1.

The step in ε imposes continuity conditions on the components of the electric displacement vector $\mathbf{D}$ and the magnetic field $\mathbf{B}$ at the bubble surface; this provides the coupling between the fields and the motion of the bubble. The latter is described by the time-dependence of the bubble radius $R(t)$ which is in the present model taken to be an externally prescribed parameter; the hydrodynamics of the bubble motion is not the concern of this letter. However, an expression for the frictional force that is due to the momentum transfer from the mechanical degrees of freedom of the bubble motion into the field is obtained as one of the end results and ought to be taken into consideration in the hydrodynamic equations of motion of the bubble [14].

The dynamics of the electromagnetic fields is classically as well as quantally described by the Hamiltonian

$$H = \int \mathrm{d}^3\mathbf{r}\left[\frac{1}{2}\left(\frac{\mathbf{D}^2}{\varepsilon} + \mathbf{B}^2\right) + \beta\frac{\varepsilon - 1}{\varepsilon}\,(\mathbf{D}\wedge\mathbf{B})_r\right]\;, \tag{2}$$

where β is the velocity of the bubble surface in units of the speed of light; the subscript r denotes the radial component of a vector with respect to the centre of the bubble. The first part of H is the usual Hamiltonian for a stationary dielectric; the second part is a motional correction which is small by virtue of β being small. This Hamiltonian has been derived from considerations of Lorentz invariance [15]; acceleration stresses have been neglected, and so have terms of order β^2 and higher.

The transition amplitude for the photon field to go from its initial vacuum to an excited state is calculated by solving the Schrödinger equation

$$\mathrm{i}\,\hbar\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi\rangle = H|\psi\rangle \tag{3}$$

with the initial condition $|\psi(t_0)\rangle = |0\rangle$. A perturbative solution of this equation to first order in the interface velocity β is called for. This poses a non-trivial problem, since the Hamiltonian H depends not only explicitly on $\dot{R}(t) \equiv \beta$ but via ε also parametrically on $R(t)$. To handle this task a judicious combination of standard perturbation theory and Pauli's theory of adiabatic approximation has been devised [16, 15]; its application yields for the transition amplitude from the initial vacuum $|0\rangle$ into a two-photon state $|k, k'\rangle$ to first order in β

$$\begin{aligned}\langle k, k'|\psi\rangle \;&=\; -\frac{1}{\omega + \omega'}\int_{t_0}^{t} \mathrm{d}\tau\;\beta(\tau)\;\mathrm{e}^{\mathrm{i}(\omega+\omega')(\tau - t)} \\ &\qquad\times\langle k, k'|\;\mathcal{F}_r\;|0\rangle_{R(\tau)}\;.\end{aligned} \tag{4}$$

where the matrix element of $\mathcal{F}_r$ has to be taken at the bubble radius $R(\tau)$. The operator $\mathcal{F}_r$ is defined by

$$\begin{aligned}\mathcal{F}_r \;&=\; -\left(1 - \frac{1}{n^2}\right)\frac{R^2}{2}\oint \mathrm{d}\Omega \\ &\qquad\times\left[\left(1 + \frac{1}{n^2}\right)D_r^2 + B_r^2 - B_\theta^2 - B_\phi^2\right]\;.\end{aligned} \tag{5}$$

One of the most intriguing results of this calculation is that $\mathcal{F}_r$ is not merely a shorthand for an integral over squared field components, but turns out to have a physical meaning; it is the radial component of the force exerted by the field onto the interface. This shows that the emission of photons by a moving dielectric is indeed intrinsically related to the zero-point fluctuations of the radiation pressure. This relation can be made even more transparent by considering the mean-square deviation of the force on the surface of a stationary bubble.

$$\begin{aligned}\triangle F_r^2 \;&=\; \langle 0|\mathcal{F}_r^2|0\rangle - \langle 0|\mathcal{F}_r|0\rangle^2 \\ &=\; \frac{1}{2}\int \mathrm{d}k \int \mathrm{d}k'\;|\langle 0|\mathcal{F}_r|k, k'\rangle|^2\;.\end{aligned} \tag{6}$$

The last expression is derived by inserting an identity operator decomposed into the complete set of photon eigenstates; as $\mathcal{F}_r$ is an operator that is quadratic in the fields only two-photon states give non-zero matrix elements. These virtual two-photon states become real when the system is perturbed, which in this case means when the dielectric starts moving. Eq. (4) reveals that the spectrum of the emitted photons is determined by the spectrum of the zero-point fluctuations of the field. As discussed above

the fluctuation-dissipation theorem underlies this fundamental interrelation, although it does not exhaust it.

In principle the transition amplitude (4) allows one to calculate all physically significant quantities concerning the radiation process. Experimentally most important is the angle-integrated spectral density

$$\mathcal{P}(\omega) = \omega^3 \int_0^T \mathrm{d}t \oint \mathrm{d}\Omega_{\mathbf{k}} \int_{-\infty}^{\infty} \mathrm{d}^3\mathbf{k}' \ |\langle k, k'|\psi\rangle|^2 \ . \qquad (7)$$

$\mathcal{P}(\omega)$ is a functional of the trajectory $R(t)$ of the bubble surface. Its direct analytical determination is hindered by the multiple occurrence of $R(\tau)$ in $\langle k, k'| \ \mathcal{F}_r \ |0\rangle_{R(\tau)}$ and by the complicated structure of this matrix element which comprises products of spherical Bessel functions and their derivatives [17]. To estimate the spectral density $P(\omega)$ one can adopt a model profile for the time-dependence of the bubble radius about the collapse

$$R^2(t) = R_0^2 - \left(R_0^2 - R_{\min}^2\right) \frac{1}{(t/\gamma)^2 + 1} \ . \qquad (8)$$

R_0 and $R_{\min}$ are the initial and minimum radii, respectively; the parameter γ describes how fast the collapse happens or, in other words, characterizes the time-scale of the turn-around of the velocity $\beta(t) \equiv \dot{R}(t)$ at $R_{\min}$. Assuming for feasibility furthermore that the bubble radius R is much greater than the wavelengths of the light emitted [17], one can derive for the spectral density

$$\mathcal{P}(\omega) = \frac{(n^2-1)^2}{64\, n^2} \frac{\hbar}{c^4\gamma} \left(R_0^2 - R_{\min}^2\right)^2 \omega^3 \, \mathrm{e}^{-2\gamma\omega} \ . \qquad (9)$$

This is a result of great significance as it shows that the spectrum of the emitted light resembles a black-body spectrum although zero-temperature quantum field theory is being dealt with. The reason for that lies in the nature of the zero-point fluctuations of the electromagnetic field. Since the Hamiltonian is quadratic in the fields, the photons are always created in pairs. The spectral density, however, is determined in a single-photon measurement which involves the tracing over the other photon in the pair; such tracing is known to make pure two-mode states look like thermally distributed single-mode states [18].

Another quantity of interest is the total energy $\mathcal{W}$ radiated during one acoustic cycle. In the short-wavelengths limit [17] one obtains

$$\mathcal{W} = \frac{(n^2-1)^2}{n^2} \frac{\hbar}{480\pi c^3} \int_0^T \mathrm{d}\tau \ \frac{\partial^5 R^2(\tau)}{\partial\tau^5} R(\tau)\beta(\tau) \ . \qquad (10)$$

From this the dissipative force acting on the moving bubble surface is seen to behave like $R^2\beta^{(4)}$ in its leading term. Such a proportionality to the fourth derivative of the velocity is also found in calculations of frictional forces on moving plane perfect mirrors [19]. The emission of photons is thus not predominantly influenced by the acceleration of the interface, which retrospectively justifies the disregard of acceleration stresses in the present model.

A reckoning based on the model trajectory (8) yields the estimate

$$\mathcal{W} = 2 \cdot 10^{-13}\,\mathrm{J} \ \text{ for } \gamma \sim 1\mathrm{fs}\,, \qquad (11)$$

which corresponds roughly to the observed number of photons. One femtosecond seems a very short time-scale for the turn-around of the velocity, but numerical calculations [15] indicate that the photon emission is substantially enhanced by resonances in the regime $kR \sim 1$, i.e. when the photon wavelengths are comparable to the bubble radius, so that in practice a turn-around time of the order of 100fs is presumably sufficient to yield the above amount of energy per burst.

In conclusion, it can be said that at this crude level of inspection the theory of vacuum radiation seems to agree remarkably well with the experimental results on sonoluminescence. The amount of the radiation and the thermal-like spectrum are returned by the theory and further numerical investigations will uncover more details. Likewise one has no difficulties explaining the shortness of the observed pulses. The pulse length is dictated by the time it takes for the zero-point fluctuations to correlate around the bubble and by the turn-around time of the velocity about the collapse; both are much shorter than 10ps. Another major point that is clarified by this theory is that there are practically no photons created in the UV and at even higher energies, as water has no appreciable polarizability there. Hence no radiation has to be absorbed by the surrounding water.

A relatively simple experiment to discriminate the present from other theories of sonoluminescence is to look for photons radiated in the X-ray transparency window of water [8]; whereas both black-body and bremsstrahlung theories predict a perceptible number of photons radiated into this window, the theory of vacuum radiation forbids them.

A second, not too difficult distinguishing experiment is to measure the angular distribution of the light emitted from an elongated rather than spherical sonoluminescent bubble achieved by anisotropic ultrasound. The present theory, unlike others, predicts an anisotropic sonoluminescence intensity, as the number of photons emitted into a certain direction is roughly proportional to the cross-section of the bubble perpendicular to this direction.

Vacuum radiation might strike one as a strange explanation for the light seen in sonoluminescence, since one often tends to think of low-energy photons as emitted by atoms. However, the present case forces one to give up this lax point of view, as atomic transitions are about a thousand times slower than a sonoluminescence pulse. On the level of quantum electrodynamics radiation comes from moving charges and it lies within one's discretion whether one groups these charges in atoms, in dipoles to make up a dielectric, or in yet another structure. For sonoluminescence it seems most convenient to think in terms of a dielectric as a whole in order to account for the cooperative response of charges to the zero-point fluctuations of the electromagnetic field.

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